

A-level
MATHS
Mechanics

Total number of marks: 44

- 11 A number of forces act on a particle such that the resultant force is $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ N

One of the forces acting on the particle is $\begin{pmatrix} 8 \\ -5 \end{pmatrix}$ N

Calculate the total of the other forces acting on the particle.

Circle your answer.

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

[1 mark]

$$\begin{pmatrix} 2 \\ -2 \end{pmatrix} \text{ N}$$

$$\begin{pmatrix} 14 \\ -8 \end{pmatrix} \text{ N}$$

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix} \text{ N}$$

$$\begin{pmatrix} -14 \\ 8 \end{pmatrix} \text{ N}$$

- 10 A vehicle is driven at a constant speed of 12 m s^{-1} along a straight horizontal road.

Only one of the statements below is correct.

Identify the correct statement.

Tick (✓) **one** box.

[1 mark]

The vehicle is accelerating

The vehicle's driving force exceeds the total force resisting its motion

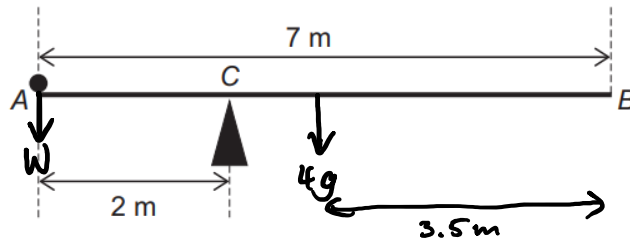
The resultant force acting on the vehicle is zero

The resultant force acting on the vehicle is dependent on its mass

13 A uniform rod, AB , has length 7 metres and mass 4 kilograms.

The rod rests on a single fixed pivot point, C , where $AC = 2$ metres.

A particle of weight W newtons is fixed at A , as shown in the diagram.



The system is in equilibrium with the rod resting horizontally.

13 (a) Find W , giving your answer in terms of g . *Anticlockwise moments = clockwise moments [2 marks] when rod in equilibrium*

Around point C:

$$W \times 2 = 4g \times 1.5$$

$$2W = 6g$$

$$W = 3g \text{ N}$$

13 (b) Explain how you have used the fact that the rod is uniform in part (a).

centre of mass of rod at its midpoint

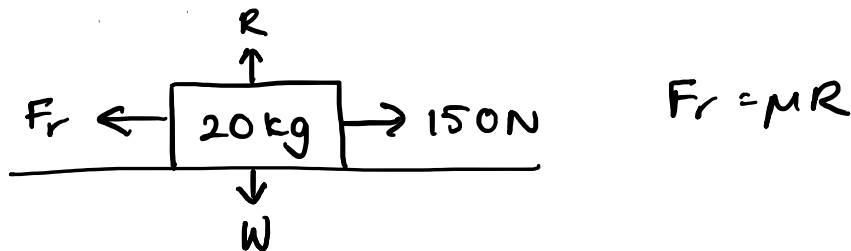
[1 mark]

13 In this question use $g = 9.8 \text{ m s}^{-2}$

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.

[3 marks]

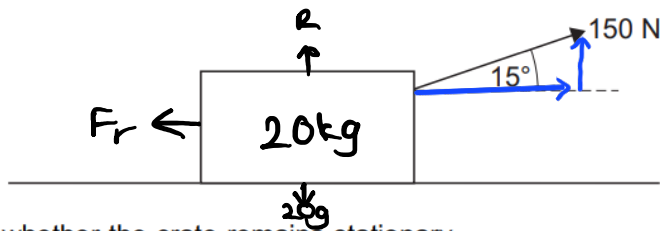


$$R = W \quad R = 20 \text{ kg} \times 9.8 \text{ m s}^{-2} = 196 \text{ N}$$

$$F_r = 0.85 \times 196 \text{ N} = 166.6 \text{ N}$$

thrust force (150 N) < friction force (166.6 N) so crate remains stationary

- 13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

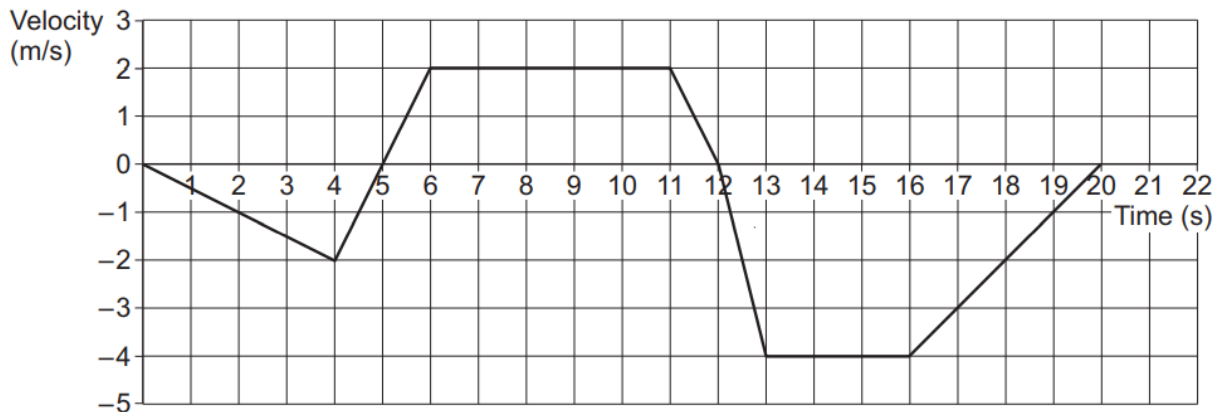
$$RN + (\sin 15^\circ) \times 150 \text{ N} = 20 \text{ kg} \times 9.8 \text{ ms}^{-2} \quad RN = 157.2 \text{ N} \quad [5 \text{ marks}]$$

$$\cos 15^\circ \times 150 \text{ N} = 144.9 \text{ N}$$

$$F_r = \mu R \quad F_r = 0.85 \times 157.2 \text{ N} = 133.6 \text{ N}$$

$144.9 \text{ N} > 133.6 \text{ N}$
 so crate moves

- 12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



- 12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

steepest gradient = greatest acceleration

$$a = \frac{v-u}{t} \quad a = \frac{(-4) - (0)}{1} = \boxed{-4 \text{ ms}^{-2}}$$

- 12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

time/s	displacement/m	
0-4	-4	} -5
4-5	-1	
5-6	1	} 5, when $t_1 = 8s$
6-11	10	
11-12	1	} 7
12-13	-2	
13-16	-12	} -9, when $t_2 = 14.25s$
16-20	-8	

- 14 At time t seconds a particle, P , has position vector \mathbf{r} metres, with respect to a fixed origin, such that

$$\mathbf{r} = (t^3 - 5t^2)\mathbf{i} + (8t - t^2)\mathbf{j}$$

- 14 (a) Find the exact speed of P when $t = 2$

[4 marks]

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (3t^2 - 10t)\mathbf{i} + (8 - 2t)\mathbf{j}$$

$$t=2 \quad (3 \times 2^2 - 10 \times 2)\mathbf{i} + (8 - 2 \times 2)\mathbf{j}$$

$$= -8\mathbf{i} + 4\mathbf{j}$$

$$\text{speed} = \sqrt{(-8)^2 + (4)^2} = (4\sqrt{5} \text{ ms}^{-1})$$

- 14 (b) Bella claims that the magnitude of acceleration of P will never be zero.

Determine whether Bella's claim is correct.

Fully justify your answer.

[3 marks]

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (6t - 10)\mathbf{i} + (-2)\mathbf{j}$$

$$a = \sqrt{(6t - 10)^2 + (-2)^2}$$

for all t values, a is at least 2 ms^{-2} minimum

as $\sqrt{0 + 4} = 2$ thus, Bella's claim is correct

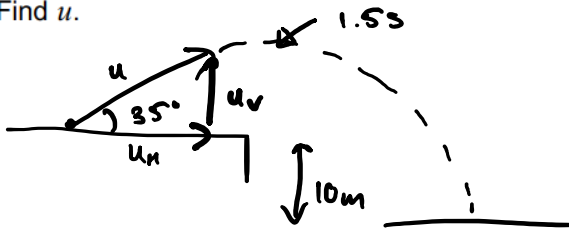
16 In this question use $g = 9.81 \text{ m s}^{-2}$

A particle is projected with an initial speed u , at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u .



$$u_v = ? \quad V = 0 \text{ m s}^{-1} \quad [3 \text{ marks}]$$

$$a = -9.81 \text{ m s}^{-2} \quad t = 1.5 \text{ s}$$

$$V = u + at$$

$$0 \text{ m s}^{-1} = u_v + (-9.81 \text{ m s}^{-2})(1.5 \text{ s})$$

$$\sin 35^\circ = \frac{u_v}{u} \quad \sin 35^\circ = \frac{14.715}{u} \quad u_v = 14.715 \text{ m s}^{-1}$$

$$u = 25.7 \text{ m s}^{-1}$$

16 (b) Find the total time that the particle is in flight.

[3 marks]

$$u = 14.715 \text{ m s}^{-1} \quad a = -9.81 \text{ m s}^{-2} \quad s = -10 \text{ m} \quad t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$-10 \text{ m} = (14.715 \text{ m s}^{-1})t + \frac{1}{2}(-9.81 \text{ m s}^{-2})(t)^2$$

$$4.905t^2 - 14.715t - 10 = 0$$

$$t = 3.57 \text{ s} \quad \text{or} \quad -0.57 \text{ s}$$

$$t = 3.57 \text{ s}$$

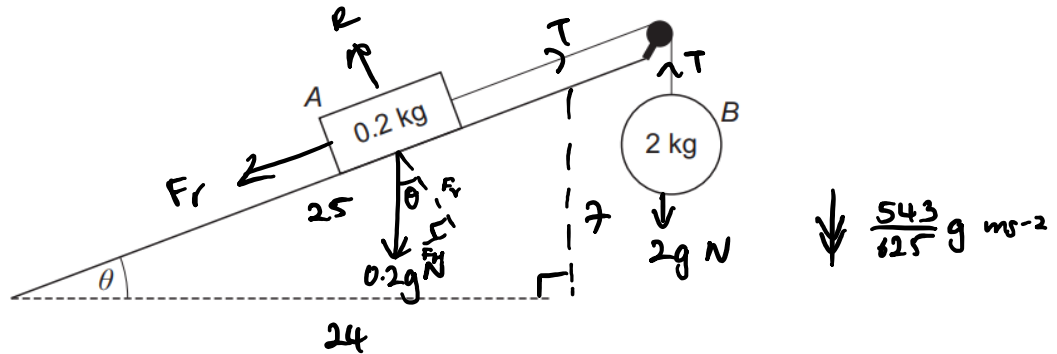
X REJECT
time cannot
be negative

18 Block A, of mass 0.2 kg, lies at rest on a rough plane.

The plane is inclined at an angle θ to the horizontal, such that $\tan \theta = \frac{7}{24}$

A light inextensible string is attached to A and runs parallel to the line of greatest slope until it passes over a smooth fixed pulley at the top of the slope.

The other end of this string is attached to particle B, of mass 2 kg, which is held at rest so that the string is taut, as shown in the diagram below.



18 (a) B is released from rest so that it begins to move vertically downwards with an acceleration of $\frac{543}{625} g \text{ ms}^{-2}$

Show that the coefficient of friction between A and the surface of the inclined plane is 0.17

[8 marks]

$$F = ma \quad 2g \text{ N} - T \text{ N} = 2 \text{ kg} \times \frac{543}{625} g \text{ ms}^{-2}$$

$$T \text{ N} = 0.2624g \text{ N}$$

$$\sin \theta = \frac{F_H}{0.2g \text{ N}} \quad \frac{7}{25} = \frac{F_H}{0.2g \text{ N}}$$

$$F_H = 0.056 g \text{ N}$$

$$F = ma \quad 0.2624g \text{ N} - 0.056 g \text{ N} - F_r \text{ N} = 0.2 \text{ kg} \times \frac{543}{625} g \text{ ms}^{-2}$$

$$F_r \text{ N} = 0.03264g \text{ N}$$

$$R = \cos \theta \times 0.2g = \frac{24}{25} \times 0.2g = 0.192g \text{ N}$$

$$F_r = \mu R \quad 0.03264g \text{ N} = \mu \times 0.192g \text{ N}$$

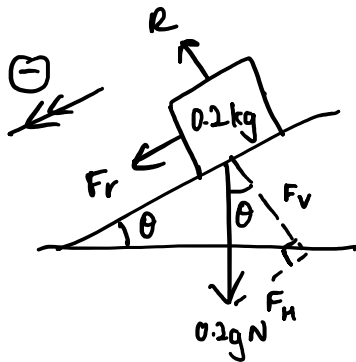
$$\mu = 0.17$$

18 (b) In this question use $g = 9.81 \text{ ms}^{-2}$

When A reaches a speed of 0.5 ms^{-1} the string breaks.

18 (b) (i) Find the distance travelled by A after the string breaks until first coming to rest.

[4 marks]



$$\sin \theta = \frac{F_H}{0.2gN}$$

$$\frac{7}{25} = \frac{F_H}{0.2gN}$$

$$F_H = 0.056gN$$

$$R = \cos \theta \times 0.2gN$$

$$= \frac{24}{25} \times 0.2gN$$

$$= 0.192gN$$

$$f_r = \mu R$$

$$F_r = 0.17 \times 0.192gN$$

$$= 0.03264gN$$

$$F = ma \quad 0N - 0.03264gN - 0.056gN = 0.2g \times a$$

$$a = -0.4432g \text{ ms}^{-2}$$

$$u = 0.5 \text{ ms}^{-1} \quad v = 0 \text{ ms}^{-1}$$

$$a = 0.1168g \text{ ms}^{-2} \quad s = ?$$

$$v^2 = u^2 + 2as \quad (0 \text{ ms}^{-1})^2 = (0.5 \text{ ms}^{-1})^2 + 2(-0.4432g \text{ ms}^{-2})s \quad \boxed{s = 0.0288 \text{ m}}$$

18 (b) (ii) State an assumption that could affect the validity of your answer to part (b)(i).

[1 mark]

no air resistance,

string doesn't obstruct block